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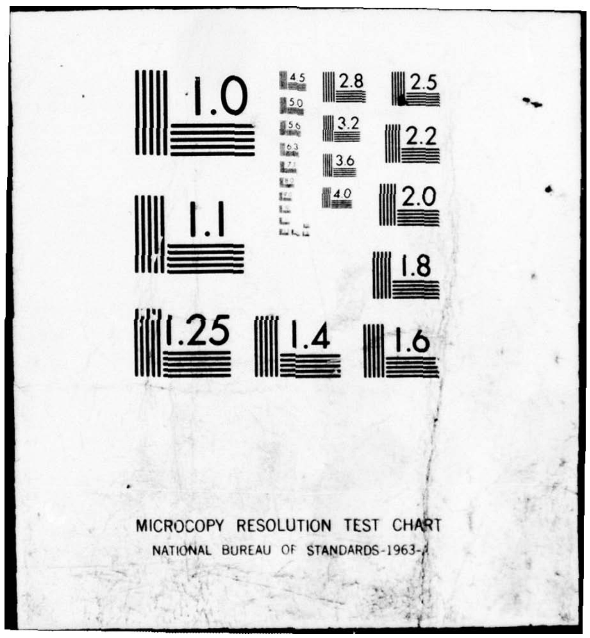
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OPTIMUM DESIGN OF FRAMES USING LINEAR PROGRAMMING TECHNIQUES

by

Walter J. Briggs[1] and Kenneth D. Willmert[2]

INTRODUCTION

In recent years, great advances have been made in the area of computerized structural analysis and optimal design. One class of optimization techniques called mathematical programming methods have been applied extensively to the design of structural systems. In most cases the resulting problems are nonlinear in the design variables and thus nonlinear programming techniques are required for the solution. The difficulty with these methods is that as the number of design variables becomes large .. on the order of 50 or more .. problems with computer costs (or time) and numerical accuracy become significant, thus limiting their applicability. The goal of the work presented here was to develop an efficient optimization technique for the design of large structures. While nonlinear programming methods are impractical for large problems, linear techniques, if applicable, can still be used. This, then, was the general approach taken in the current research.

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The use of linear programming methods in the design of trusses has been investigated by a number of researchers. Dorn, Gomory, and Greenberg [5] have shown that the optimum design of a truss, subjected to stress limitations, can be formulated as a linear programming problem. The authors considered the geometry of the structure to be fixed and took the member areas as design variables. All trusses obtained from their approach were determinate, since only conditions of equilibrium were imposed. Farshi and Schmit [6] developed a technique for the optimization of trusses based on the force method of analysis. Initially only stress constraints and the conditions of equilibrium were considered, and then in an iterative manner the conditions of compatibility were added until a suitable design was obtained. In studies by Reinschmidt, Norabhoompipat and Russel [12,13], an approach similar to that of [6] was used. One conclusion from their work was that the approximate solution found by satisfying only the equilibrium conditions was close to the optimum design obtained by enforcing the conditions of compatibility as well. In an investigation by Schmit and Farshi [16] an efficient technique for the optimization of trusses subjected to both stress and displacement constraints was developed. In order to obtain the optimum design a sequence of linear programming problems were solved. The optimization algorithm was based on the method of inscribed hyperspheres [1]. High efficiency was achieved by using several approximation concepts such as design variable linking, temporary deletion of noncritical constraints, and Taylor series expansions for response variables in terms of the design variables. The authors concluded

from this study that the use of approximation concepts along with linear programming techniques could lead to a new generation of efficient and practical optimization procedures.

The design of frames has also been formulated such that linear programming techniques can be applied. Moses [11] performed successive Taylor series expansions to obtain linear design constraints. An iterative approach, based on the simplex method, was used in solving for the final design. In the work of Rubinstein and Karagozian [14], plastic hinge theory was used to generate the equations of constraint on the moment carrying capacity of the members. The design criteria was safety against collapse and a limitation on lateral deflections in the elastic range. Romstad and Wang [15] used another approach to formulate the optimization of frames as a linear programming problem. Linear behavior constraints on stresses and displacements were developed by initiating local changes in the design parameters and then determining the resulting stress and displacement redistributions. A design cycle in this formulation involved analyzing the structure using assumed design variables via the displacement method of structural analysis. These results were then used to generate the linear behavior constraints which comprised the linear programming problem. The solution of this linear programming problem resulted in a lower weight design with modified design variables, which completed the cycle.

In the work presented here, the optimum frame design problem is formulated in a new way such that linear programming methods can more efficiently be applied. Stress constraints, as well as conditions

of equilibrium and compatibility are considered. The approach does not require a separate time consuming analysis phase to determine member stresses or forces (as does the Romstad's and Wang's method discussed previously), since these quantities are variables within the linear formulation, and consequently are determined as part of the linear programming solution. An additional feature of the technique presented is that the linear design constraints are obtained without the use of Taylor series expansions or various other approximations. In the initial formulation, some of the design constraints were nonlinear, but by holding a small number of parameters constant all constraints become linear in the design variables. The solution to the problem is thus obtained in an iterative manner, i.e., initial values for the parameters are selected and the linear problem solved; then the parameters in the nonlinear constraints are revised and the process repeated. This procedure continues until the parameters converge to their optimal values. Although the technique is iterative, no series expansions like those of [11] and [15], which can be very inaccurate, are required. Also, efficient linear programming methods have been developed in this work so that each iteration of the technique requires little computational time.

FORMULATION

The problem considered can be stated as follows: given a framed structural system with fixed configuration subject to externally applied loads, find the design, i.e. the cross-sectional size of the members, such that the weight of the system is minimized subject to

limitations on the stresses within the elements. In this investigation, the loading on the structure was restricted to a single loading condition with all forces and moments concentrated at the nodes. Several other standard assumptions were made as well. All structural systems were assumed to behave in a linear elastic manner (see Ref. [2]), and the stress in the elements was taken as a linear combination of both the axial and bending stresses (see the Manual of Steel Construction [10] section 1.6). All frame members were assumed to be rolled wide flange steel shapes, with section properties, i.e. cross-sectional area, moment of inertia, and section modulus, approximated as linear dependent quantities. An approach similar to this was used by Brown and Ang [3].

The objective function for the general problem is thus:

$$w = \rho \sum_{j=1}^{n} L_j A_j \qquad (1)$$

which must be minimized, where A_j and L_j are the cross-sectional area and length of the j^{th} element respectively, ρ is the weight per unit volume, and n is the total number of elements in the system. The stress constraints can be written in the form:

$$\frac{|F_j|}{\sigma A_j} + \frac{|M_j|}{K\sigma A_j} \leq 1 \qquad j=1,2,\ldots,n \qquad (2)$$

where σ is the allowable stress, K represents a constant of proportionality between the section modulus and cross-sectional area, F_j is the axial force in the j^{th} member and M_j is the maximum moment along the length of the member. Since it was assumed that the external forces and moments are applied only at the nodes of the structure,

then the maximum stress will occur at one of its ends. Which end possesses the maximum stress, however, is unknown, thus constraints are written for both the i and k ends:

$$\frac{|F_j|}{\sigma} + \frac{|M_{ji}|}{\sigma K} \leq A_j$$
$$\frac{|F_j|}{\sigma} + \frac{|M_{jk}|}{\sigma K} \leq A_j \qquad j=1,2,\ldots,n \qquad (3)$$

The equilibrium equations for the structure can be obtained by summing forces and moment at the nodes producing:

$$\sum_{j=1}^{n} L_{ji}\,\overline{S}_j = \overline{R}_i \qquad i=1,2,\ldots,m \qquad (4)$$

where m is the total number of free nodes, $\overline{R}_i$ is a vector of externally applied forces and moments at node i, $\overline{S}_j$ is a vector of fundamental member forces for the j^{th} member:

$$\overline{R}_i = \begin{bmatrix} R_{ix} \\ R_{iy} \\ M_{iz} \end{bmatrix} \qquad \overline{S}_j = \begin{bmatrix} F_j \\ M_{ji} \\ M_{jk} \end{bmatrix} \qquad (5)$$

and L_{ji} is a transformation matrix:

$$L_{ji} = \begin{bmatrix} -t_{j1} & t_{j2}/L_j & t_{j2}/L_j \\ t_{j2} & t_{j1}/L_j & t_{j1}/L_j \\ 0 & 1 & 0 \end{bmatrix} \qquad (6)$$

where $t_{j1} = (X_k - X_i)/L_j$ and $t_{j2} = (Y_i - Y_k)/L_j$. The (X_i, Y_i) and (X_k, Y_k) are the coordinates of the end points of the j^{th} member.

The compatibility conditions can be obtained by considering a general element j as shown in Figure 1. Position 1-2 corresponds to the initial position before deformation, while the final position 1'-2" is composed of a combination of a rigid body displacement

from 1-2 to 1'-2' and a deformation from 1'-2' to 1'-2". The relationship between local deformations and fundamental forces is:

$$\begin{bmatrix} U_k - U_i \\ V_k - V_i - \theta_i L_j \\ \theta_k - \theta_i \end{bmatrix} = \begin{bmatrix} \frac{L_j}{EA_j} & 0 & 0 \\ 0 & -\frac{L_j^2}{3EI_j} & \frac{L_j^2}{6EI_j} \\ 0 & -\frac{L_j}{2EI_j} & \frac{L_j}{2EI_j} \end{bmatrix} \begin{bmatrix} F_j \\ M_{ji} \\ M_{jk} \end{bmatrix} \tag{7}$$

Transforming the local deformations to global coordinates, equation (7) can be written as:

$$C_j \overline{U}_j = D_j \overline{S}_j \qquad j=1,2,\ldots,n \tag{8}$$

where $\overline{U}_j$ is a vector of horizontal and vertical deformations and rotations at the ends of the member, D_j is the matrix defined in equation (7), and C_j is:

$$C_j = \begin{bmatrix} t_{j1} & -t_{j1} & -t_{j2} & t_{j2} & 0 & 0 \\ t_{j2} & -t_{j2} & t_{j1} & -t_{j1} & 0 & -L_j \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{9}$$

The relation between deformations and fundamental member forces for the entire structure can be obtained by combining the element equations (8) and applying the boundary conditions producing:

$$C\,\overline{U} = D\,\overline{S} \tag{10}$$

where $\overline{U}$ is a vector of global displacements of the free nodes in the system, C is a matrix of direction cosines, D is a matrix containing the D_j matrices on the diagonal and zeros everywhere else, and $\overline{S}$ is a vector of all the fundamental forces in the structure. The size of C is 3n by 3m and D is 3n by 3n, where n is the total number of elements in the structure and m is the number of free nodes.

The compatibility equations are formed by eliminating the displacements from the last 3n - 3m equations of (10). This is achieved by performing a Gaussian elimination on the matrix C such that the upper 3m by 3m matrix becomes upper triangular and the lower 3n - 3m by 3m submatrix contains all zeros. If these same operations are performed on the matrix D, then the deformations will be eliminated from the last 3n - 3m equations in (10). These are the desired compatibility equations of the form:

$$H \bar{S} = 0 \tag{11}$$

The matrix H, however, is a function of the member properties, in particular, the areas and moments of inertia which are variables in the design problem.

The initial formulation is now complete. Taking the fundamental member forces and areas as variables the objective function (1) and equilibrium constraints (4) are linear. However, the stress constraints (3) are nonlinear because of the absolute values and the compatibility equations (11) are nonlinear since H is a function of the areas and moments of inertia. Thus linear programming techniques can not be used on the problem in its current form.

TRANSFORMATION TO LINEAR FORM

The absolute values in the stress constraints (3) can easily be eliminated by replacing each of the inequalities in (3) by four constraints of the form:

$$
\begin{aligned}
A_j - \frac{F_j}{\sigma} - \frac{M_{ji}}{K\sigma} &\geq 0 \\
A_j - \frac{F_j}{\sigma} + \frac{M_{ji}}{K\sigma} &\geq 0 \\
A_j + \frac{F_j}{\sigma} - \frac{M_{ji}}{K\sigma} &\geq 0 \\
A_j + \frac{F_j}{\sigma} + \frac{M_{ji}}{K\sigma} &\geq 0
\end{aligned}
\tag{12}
$$

and similarly for the k^{th} end of the element. The four inequalities (12) impose the same stress restriction as does the original one involving absolute values, but are now linear in the areas and fundamental member forces.

The compatibility constraints (11) cannot easily be converted to linear form; however, they can be written such that the components of the *H matrix* are *linear functions of moment* of inertia ratios:

$$\beta_j = \frac{I_1}{I_j} \qquad j=2,3\ldots,n \tag{13}$$

which become the only variables in this matrix. For example, the H matrix for the three member portal frame shown in Figure 2 is:

$$
H = \begin{bmatrix}
0 & \frac{L_1^2}{3} & -\frac{L_1^2}{6} & L_2K'\beta_2 & 0 & 0 & 0 & \frac{L_3^2\beta_3}{6} & \frac{L_3^2\beta_3}{3} \\
L_1K' & -\frac{L_1L_2}{2} & \frac{L_1L_2}{2} & 0 & -\frac{L_2^2\beta_2}{3} & \frac{L_2^2\beta_2}{6} & -L_3K'\beta_3 & 0 & 0 \\
0 & -\frac{L_1}{2} & \frac{L_1}{2} & 0 & -\frac{L_2\beta_2}{2} & \frac{L_2\beta_2}{2} & 0 & -\frac{L_3\beta_3}{2} & \frac{L_3\beta_3}{2}
\end{bmatrix}
\tag{14}
$$

where the fundamental member force vector is ordered as:

$$\bar{S}^T = [F_1 \; M_{11} \; M_{12} \; F_2 \; M_{22} \; M_{23} \; F_3 \; M_{33} \; M_{34}] \tag{15}$$

The constant K' is the assumed ratio of the moment of inertia to the cross-sectional area for rolled wide flange steel shapes. The ratios β_j are, of course, functions of the areas, since:

$$\beta_j = \frac{I_1}{I_j} = \frac{K'A_1}{K'A_j} = \frac{A_1}{A_j} \tag{16}$$

but if these ratios are held fixed, then the compatibility constraints will be linear in the fundamental member forces. Thus the entire formulation will be a linear programming problem. This suggests an iterative approach where the ratios β_j are initially selected, for example all set to one, and the linear programming problem solved for the areas and internal forces. Using these areas and equation (16), the ratios are changed and the linear problem solved again. The process is repeated until the solution of the linear programming problem from one iteration to the next is approximately the same. By solving several examples, it appears that this technique does converge to the optimal solution, and in relatively few iterations.

SIMPLEX METHOD OF SOLUTION

In order to solve the linear programming problem at each iteration, the revised simplex method was used [4,8]. This technique, of course, requires all variables to be greater than or equal to zero, which is not the case for the internal forces in the current formulation. Thus a transformation of variables was performed using three of the stress constraints for each member. Equivalent forces were defined as:

$$Z_{j1} = A_j - \frac{F_j}{\sigma} - \frac{M_{ji}}{K\sigma}$$
$$Z_{j2} = A_j + \frac{F_j}{\sigma} - \frac{M_{ji}}{K\sigma} \qquad (17)$$
$$Z_{j3} = A_j + \frac{F_j}{\sigma} + \frac{M_{jk}}{K\sigma}$$

which must be greater than or equal to zero in order for the stress constraints to be satisfied. Equations (17) were solved for the internal forces F_j, M_{ji}, and M_{jk} in terms of the equivalent forces Z_{j1}, Z_{j2}, and Z_{j3} and substituted into the remaining formulation. This transformation has the advantage that three of the eight stress constraints for each member are eliminated, while the number of variables in the problem remains the same.

Experience has shown that the areas of the members cannot be permitted to approach zero, since this results in extremely large moment of inertia ratios β_j which produces instabilities. Thus a lower bound Q was placed on the areas, and a transformation performed so as not to increase the number of constraints. Equivalent areas A'_j were defined as:

$$A'_j = A_j - Q \geq 0 \qquad j=1,2,\ldots,n \qquad (18)$$

Using equation (18) the areas A_j were then replaced with A'_j in the formulation, where $A'_j \geq 0$. The result is a linear programming problem in the equivalent forces Z_{j1}, Z_{j2}, Z_{j3} and equivalent areas A'_j for each member, which can easily be solved using the standard revised simplex method.

After solving the linear programming problem with an initial set of moment of inertia ratios, these ratios are changed, as described previously, and a new linear problem is formed. Instead of using the standard revised simplex method, with a phase I and

phase II, to solve the new problem, a modified approach was used which greatly improved efficiency. The technique was to assume that the same variables which were basic at the optimum of the old problem remained basic for the new problem. First it was necessary to calculate the inverse of the basis matrix B for the new problem. But since only a very few constraints changed from the old problem to the new, i.e. only the compatibility constraints, and since the inverse was available from solving the old problem, the new inverse was calculated from the old one using the product form of the inverse. This is similar to the procedure used in the revised simplex method itself in changing the basis. The only difference is that rows of the matrix are changed rather than columns. If $\bar{a}_r$ is a vector corresponding to the r^{th} row of the basis matrix which is changed from the old to the new problem, then:

$$B_r^{-1} = B^{-1} T \tag{19}$$

where B^{-1} is the old inverse, B_r^{-1} is the new inverse after changing the r^{th} row alone, and T is the matrix:

$$T = \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \\ \vdots \\ \bar{\eta} \\ \vdots \\ \bar{e}_\ell \end{bmatrix}$$

where ℓ is the number of constraints in the linear programming problem. The rows $\bar{e}_i$ are vectors containing all zeros except for a one in the i^{th} position. The row $\bar{\eta}$ is:

$$\bar{\eta} = [- \frac{\tau_1}{\tau_k} - \frac{\tau_2}{\tau_k} \dots + \frac{1}{\tau_k} \dots - \frac{\tau_\ell}{\tau_k}]$$

where the k^{th} variable is basic in the r^{th} constraint, and τ_1 to τ_ℓ are components of the vector $\bar{\tau}$ which can be calculated from:

$$\bar{\tau} = \bar{a}_r^T B^{-1}$$

Equation (19) is applied successively for each row of the coefficient matrix corresponding to the compatibility constraints. The result is the inverse of the basis matrix for the new problem.

Once B^{-1} is determined the values of the basic variables $\bar{x}_B$ can be calculated. If all of the variables are greater than or equal to zero, then the standard revised simplex method can be used to obtain the optimal solution of the new problem. However, if one or more of these variables is negative then a special pivotal operation is performed. The variable to leave the basis is selected to be the one which is most negative, i.e.

$$x_{Br} = \max_{x_{Bi} < 0} |x_{Bi}| \tag{20}$$

The variable x_j to enter the basis is selected corresponding to:

$$\frac{c_j - z_j}{a_{rj}} = \max \left\{ \frac{c_i - z_i}{a_{ri}} \,\middle|\, \frac{c_i - z_i}{a_{ri}} \leq \min \frac{c_k - z_k}{a_{rk}} \right\} \tag{21}$$

for $c_i - z_i < 0$, $a_{ri} < 0$, $c_k - z_k \geq 0$ and $a_{rk} > 0$, or if this does not yield a variable x_j, then corresponding to:

$$\frac{c_j - z_j}{a_{rj}} = \max \left\{ \frac{c_i - z_i}{a_{ri}} \right\} \tag{22}$$

for $c_i - z_i \geq 0$ and $a_{ri} < 0$, or if this does not yield a variable x_j, then finally corresponding to:

$$\frac{c_j - z_j}{a_{rj}} = \min \left\{ \frac{c_i - z_i}{a_{ri}} \right\} \tag{23}$$

for $c_i - z_i < 0$ and $a_{ri} < 0$. The quantities a_{ij} are the elements of the coefficient matrix for the linear programming problem and $c_i - z_i$ are the coefficients of the objective function as the simplex method proceeds. If x_j can be selected corresponding to the first criterion (21), then pivoting will result in x_j becoming positive, the negative variable x_{Br} being driven to zero, the objective function decreasing, and all positive $c_k - z_k$ remaining positive. The second criterion (22) is the normal approach used for pivoting in the dual simplex method. It can easily be shown that selecting a variable x_j to enter the basis is always possible using one of the three criteria (21), (22) or (23).

This pivotal procedure is repeated until all negative variables have been eliminated, at which point the normal revised simplex method is used to obtain the optimal solution of the new problem. Experience has shown that this procedure is very efficient in solving the new problem, in most cases requiring only a few pivotal operations.

EXAMPLES

The technique developed in this study is based upon the presumption that the moment of inertia ratios β_j will eventually converge to their optimum values. The purpose of the first three examples was to show that the linear approach could indeed determine the optimal design.

The results of the linear method were checked by solving the same problems via a nonlinear programming technique. The nonlinear approach consisted of the Fiacco and McCormick penalty function [7] along with the Hooke and Jeeves direct search algorithm [9]. The penalty function used was:

$$p(\bar{x},r) = f(\bar{x}) + r \sum_{j=1}^{2n} \frac{1}{g_j} \tag{24}$$

where $p(\bar{x},r)$ is the penalty function, $f(\bar{x})$ is the objective function, g_j is the j^{th} inequality constraint, and r is the penalty parameter. The cross-sectional areas A_j of the members were used as design variables. The original stress inequalities (3) and lower bounds on the areas (18) were the only constraints used. At each step, the fundamental member forces necessary to evaluate the stress constraints were calculated using an appropriate analysis technique.

All design examples incorporated the following constants:

$Q = 5.0$ in^2 (32.3 cm^2) lower bound on the areas

$\sigma = 24$ ksi (165,600 kN/m^2) allowable stress

$K = 9.0$ section modulus/area

$K' = 75.0$ moment of inertia/area

Also in the linear approach, the optimum design was assumed to have been obtained when the change in the objective function in two successive iterations was less than 0.05 percent.

Shown in Figure 2 is the first design problem considered, which is a portal frame subjected to a set of nodal loads. The moment of inertia ratios β_2 and β_3 were initialized at 1.0 in the linear technique, and the initial member areas for the nonlinear technique were taken as 90.0 sq. in. (580.5 cm^2). Table 1A and 1B show the progression of the linear and nonlinear techniques, respectively, towards the optimum design. In the linear approach, we define a cycle as the process of obtaining the optimum of the linear programming problem for a given set of moment of inertia ratios, while the minimization of the penalty function for a particular value of the penalty parameter r is

considered a cycle in the Fiacco and McCormick routine. For this example, the number of cycles necessary for convergence was six for the linear approach as compared to seven for the nonlinear method of solution. As can be seen from Tables 1A and 1B the optimum values of the cross-sectional areas obtained from both methods were virtually identical. The objective function, which was taken as the volume of the structure rather than weight, was 15,525.0 cu. in. (254,610 cm^3) from the linear approach, and 15,526.0 cu. in. (254,626 cm^3) from the nonlinear solution. The total time of execution was 00:00:36 (hours: minutes:seconds) for the linear technique versus 00:00:42 for the Fiacco and McCormick routine, which indicates only a slight increase in efficiency for the linear method.

The four member frame shown in Figure 3 was used as the second example. Again, the starting point for the linear technique consisted of all the moment of inertia ratios being 1.0, while the initial areas for the nonlinear approach were taken as 75.0 sq. in. (483.7 cm^2) throughout. Tables 2A and 2B reveal the route both methods followed as each proceeded towards the optimum design. As in the first example, the final design obtained from the linear and nonlinear techniques were virtually the same with the objective function being 22,871.0 cu. in. (375,084 cm^3) from the linear solution as compared to 22,892.0 cu. in. (375,429 cm^3) from the nonlinear approach. For this example the number of cycles necessary for convergence was eighteen in the linear technique versus eight in the nonlinear approach. Although the number of cycles was larger for the linear solution, the total execution time was much less, with the linear technique taking 00:00:45 while the Fiacco and

McCormick routine required 00:03:18. Thus, the linear programming approach exhibited a substantial savings in computational effort over the nonlinear routine.

The last example used for comparing the linear technique against the nonlinear routine was the ten member frame shown in Figure 4. The moment of inertia ratios for all the members were again initialized at 1.0 in the linear approach, while the initial member areas for the nonlinear technique were 125.0 sq. in. (806.3 cm^2) throughout. Tables 3A and 3B show the progression of both methods towards the optimum design. The final structures from the two techniques are similar. A slightly lower value of the objective function was obtained from the linear approach, being 65,647.0 cu. in. (1,076,611 cm^3) as compared to 66,191.0 (1,085,532 cm^3) from the Fiacco and McCormick routine. This difference is due to the fact that the linear technique yielded a design with all members fully stressed, whereas the nonlinear approach satisfied the convergence criteria before reaching the fully stressed condition. Computational time of the two methods varied drastically, with the linear approach taking 00:02:57 versus 01:42:32 for the Fiacco and McCormick routine. This example demonstrates that the nonlinear approach is very computationaly inefficient when applied to a problem with large numbers of design variables.

Shown in Figure 5 is a plot of computational time versus the number of members in the system for both of the methods discussed. Also included in the plot corresponding to the linear technique is the computational time for the 15 member frame presented as the next example. As can be seen, the linear technique, compared to the

particular nonlinear method used in this work, required much less computational time for larger structures.

A fourth example was examined to show the effect of the initial moment of inertia ratios on the final design. Shown in Figure 6 is a 15 member two bay three story frame subjected to a single nodal load condition. This frame was designed using three unique starting points, which will be referred to as Cases I, II, and III. In Case I, all of the moment of inertia ratios were initalized as 1.0, while Case II started with a value of 5.0 for all of the member ratios, and Case III, which used a bad starting point, was initiated with values of the ratios which differed greatly from the optimal values obtained in the first two cases. Table 4 shows the initial and final moment of inertia ratios, final member areas, and the values of the objective function obtained from the three cases. The final designs are almost identical with the optimum value of the objective function being 132,805 cu. in. (2,178,002 cm^3), 132,843 cu. in. (2,178,625 cm^3), and 132,846 cu. in. (2,178,674 cm^3) for Cases I, II, and III respectively. The number of design cycles and computational times were: 7 cycles and 00:08:21, 13 cycles and 00:11:32, and 17 cycles and 00:13:31 for the three cases. Thus, as expected, computational times increased for starting points which were further from the optimal design, but the increase is not overwhelming.

The last example considered was the 25 member frame shown in Figure 7. Again two different starting points were used, with both converging to approximately the same final design as shown in Table 5.

It is interesting to note that eight of the member areas in the optimal design were at their lower bound of 5 sq. in. (32.3 cm^2). Thus, if some or all of these members were eliminated from the structure and the optimal design obtained again, it would likely have a lower weight. The elimination of the members might, however, be influenced by other considerations, such as stability, deflections, etc.

CONCLUSIONS

The linear technique presented in this paper appears to be an effective method of optimizing a framed structure. Although no proof of convergence has been developed, the method was able to determine the optimum design for all examples considered and all initial starting moment of inertia ratios. It also seems to be a very efficient method. For larger problems, the computational time for the linear technique was much less than for the nonlinear method. Although other nonlinear programming techniques, or improvements in the one used, could lead to decreased computational times, it is felt from previous experience that most nonlinear methods could not be computationally competitive. Also, nonlinear techniques have other problems associated with them, such as relative minima, premature termination, etc. which do not seem to occur for the linear method.

Although only simple loading conditions and stress constraints were considered in this work, it should be easy to extend the method to include multiple loading conditions, and non-nodal loads (as long

as finding the maximum stress in the members does not result in complex nonlinear constraints). Likewise displacement constraints can easily be added. This can be done by continuing the Gaussian elimination on the force-deflection equations (10) used to obtain the compatibility constraints (11). The goal would be to reduce the upper 3m by 3m matrix of C to the identity matrix, thus producing an expression for the displacements:

$$\bar{U} = G\,\bar{S} \tag{25}$$

where G is the upper 3m by 3n part of the matrix D of equation (10) after the Gaussian elimination has been performed on it. Then displacements constraints of the form:

$$G\,\bar{S} \leq \bar{U}_{max} \tag{26}$$

could be included on the problem. These can be written such that the moment of inertia ratios β_j are the only variables multiplying the fundamental forces $\bar{S}$; therefore, the displacement constraints can be handled in the same way as the compatibility conditions were satisfied in method presented here.

With these extensions, it is believed that the linear programming technique developed in this research can be an effective method of solving optimal frame design problems.

ACKNOWLEDGMENT

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APPENDIX I. - REFERENCES

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APPENDIX II. - NOTATION

The following symbols are used in this paper:

A_j = cross-sectional area of the j^{th} member;

A_j' = equivalent area;

a_{ij} = elements of the coefficient matrix;

B = basis matrix;

C = matrix of direction cosines;

$c_j - z_j$ = objective function coefficients;

D = force matrix;

$\bar{e}_i$ = unit vectors;

F_j = axial force in jth member;

g_j = inequality constraint;

H = compatibility matrix;

I_j = moment of inertia in j^{th} member;

K = section modulus/area;

K' = moment of inertia/area;

L_j = length of j^{th} member;

L_{ji} = force transformation matrix;

M_j = moment in the jth member;

m = number of joints in the structure;

n = number of members in the structure;

p = penalty function;

Q = lower bounds on the areas;

$\bar{R}_i$ = vector of external loads at i^{th} joint;

r = penalty function parameter;

$\bar{S}$ = vector of internal loads;

T = matrix used to calculate B inverse;

t_j = direction cosines of j^{th} member;

$\bar{U}$ = vector of nodal displacements;

w = weight of structure;

$\bar{x}$ = vector of variables;

Z_j = equivalent forces in j^{th} member;

β_j = moment of inertia ratio for j^{th} member;

$\bar{\eta}$ = vector used to calculate B inverse;

ρ = weight per unit volume;

σ = allowable stress;

$\bar{\tau}$ = vector used to calculate B inverse.

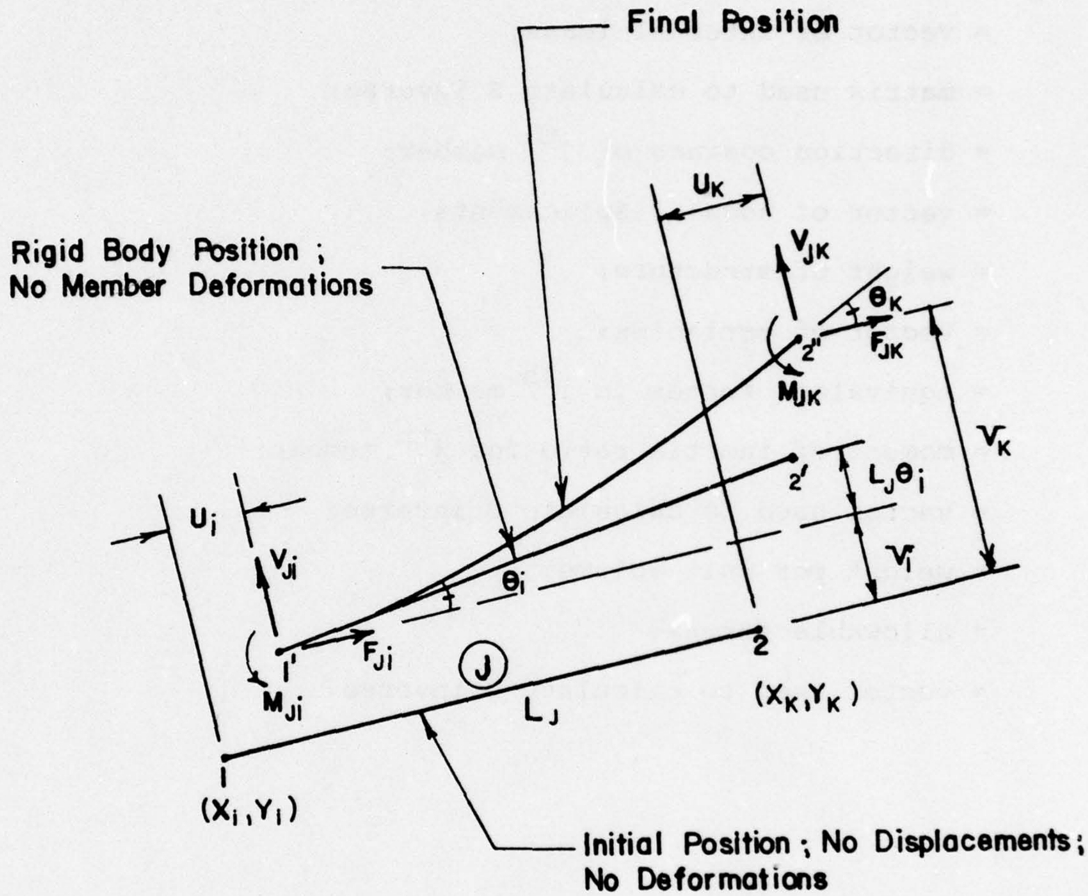

Figure 1: Relative Displacements of a Frame Element

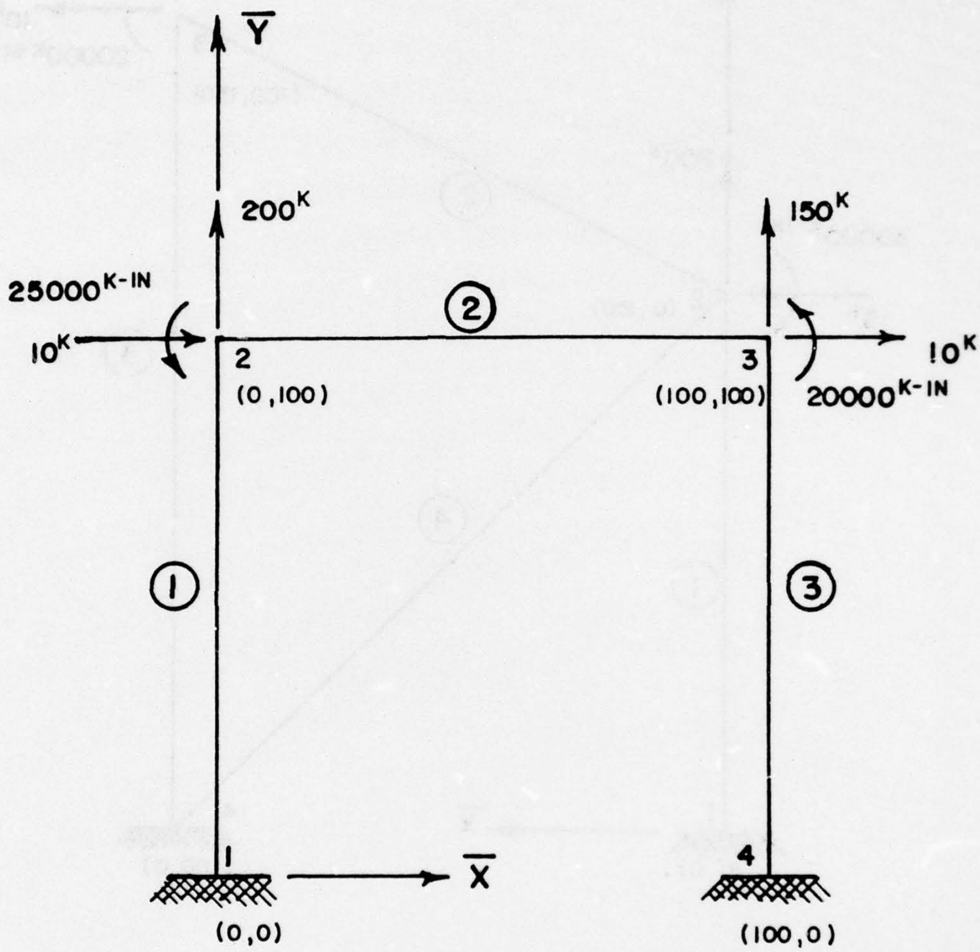

Figure 2: Three Member Frame (1 kip = 4.45 kN; 1 in = 25.4 mm; 1 kip-in = 113 N·m)

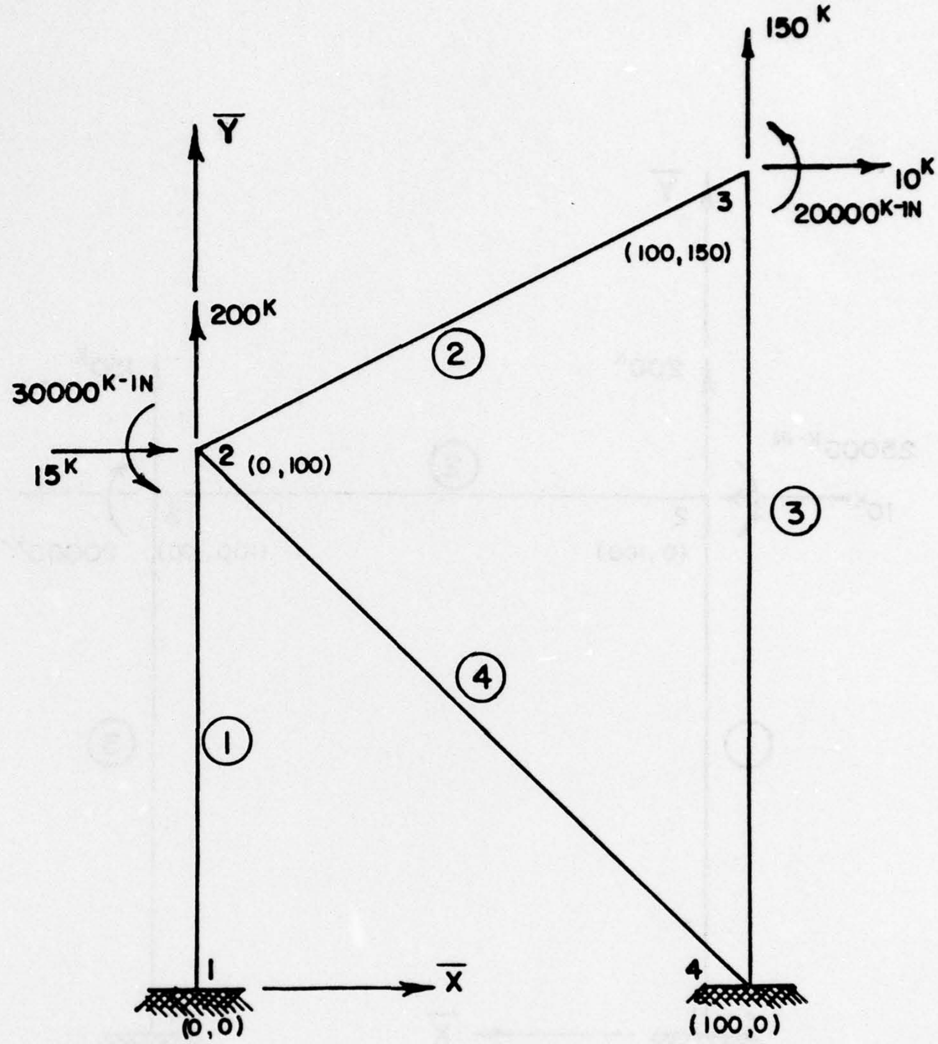

Figure 3: Four Member Frame (1 kip = 4.45 kN; 1 in = 25.4 mm; 1 kip-in = 113 N·m)

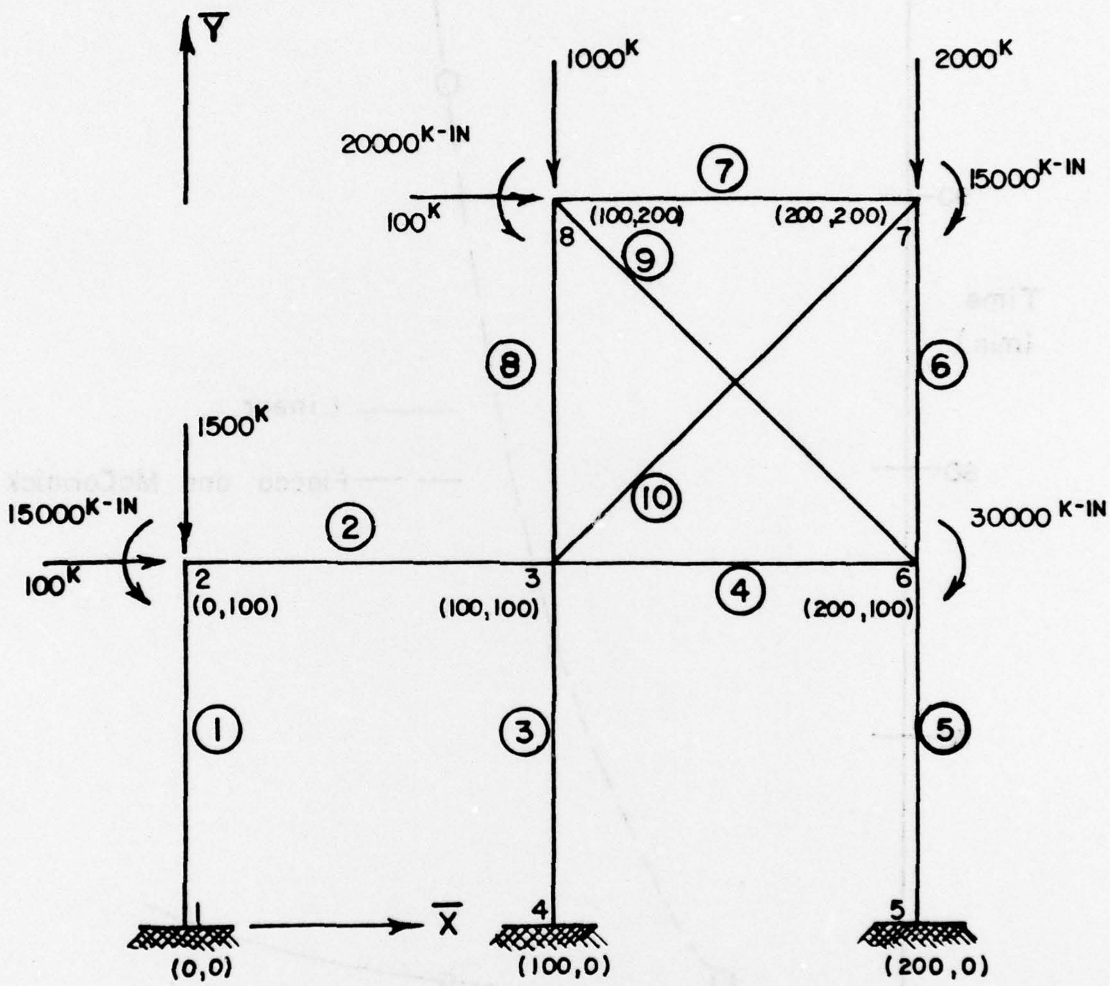

Figure 4: Ten Member Frame (1 kip = 4.45 kN; 1 in = 25.4 mm; 1 kip-in = 113 N·m)

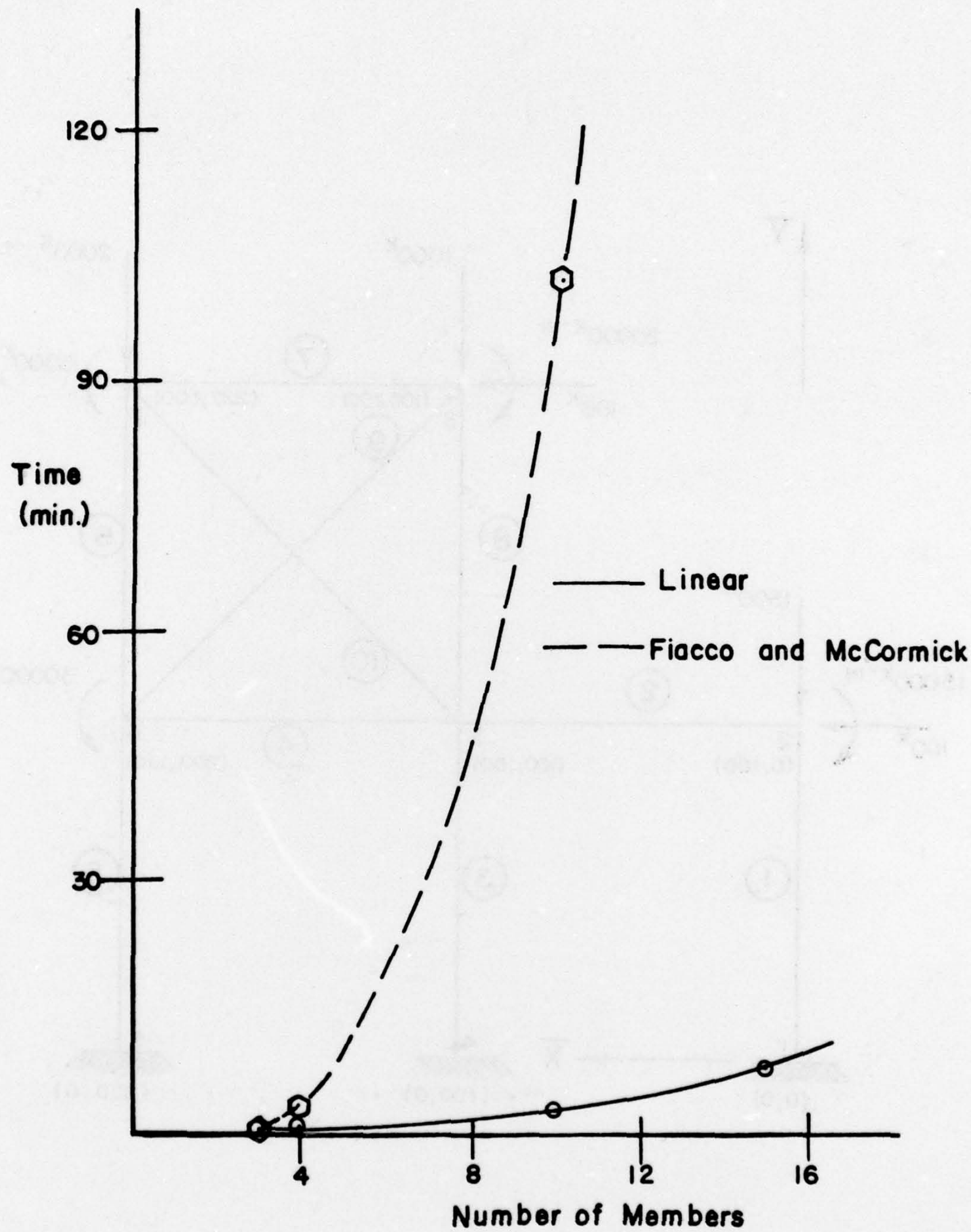

Figure 5: Computational Time versus Number of Members

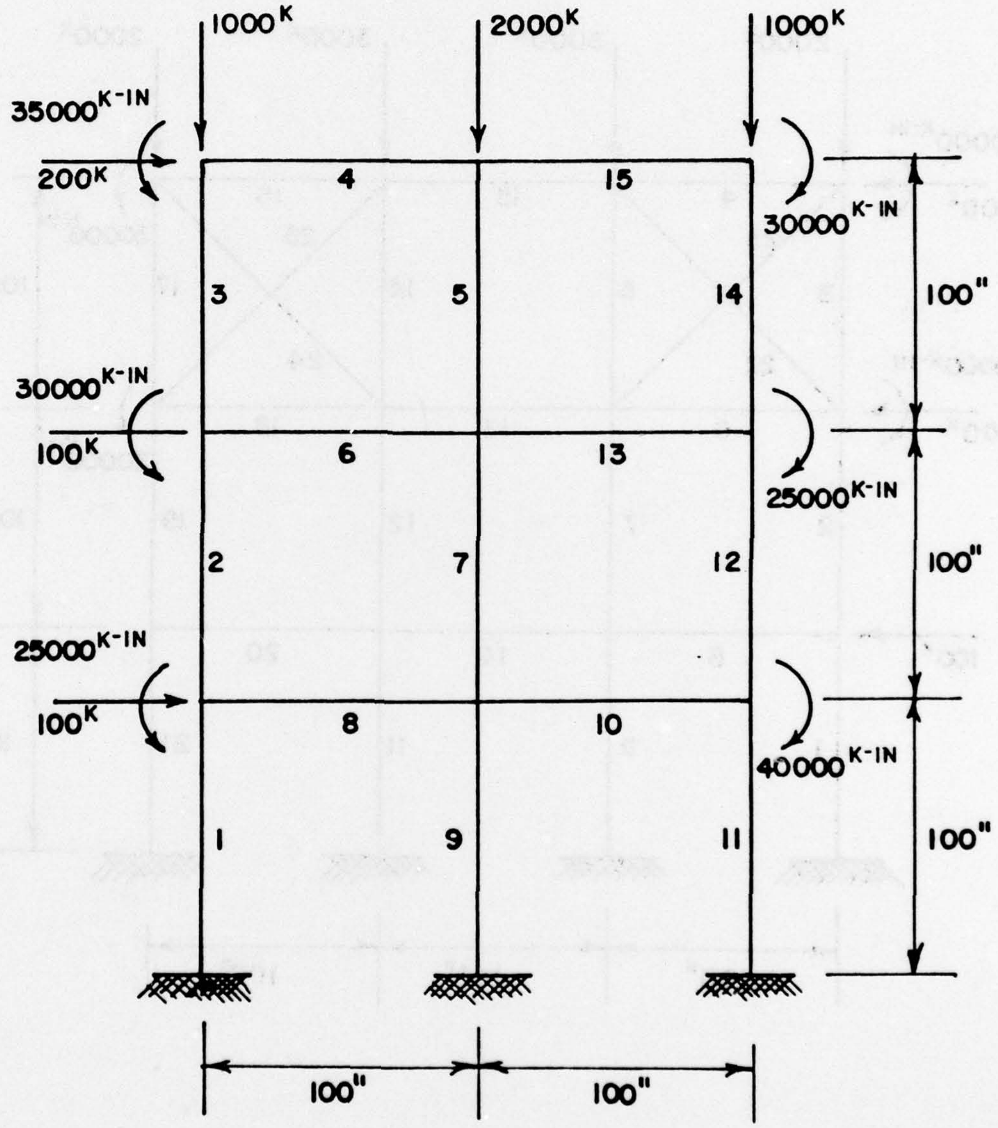

Figure 6: 15 Member Frame (1 kip = 4.45 kN; 1 in = 25.4 mm; 1 kip-in = 113 N·m)

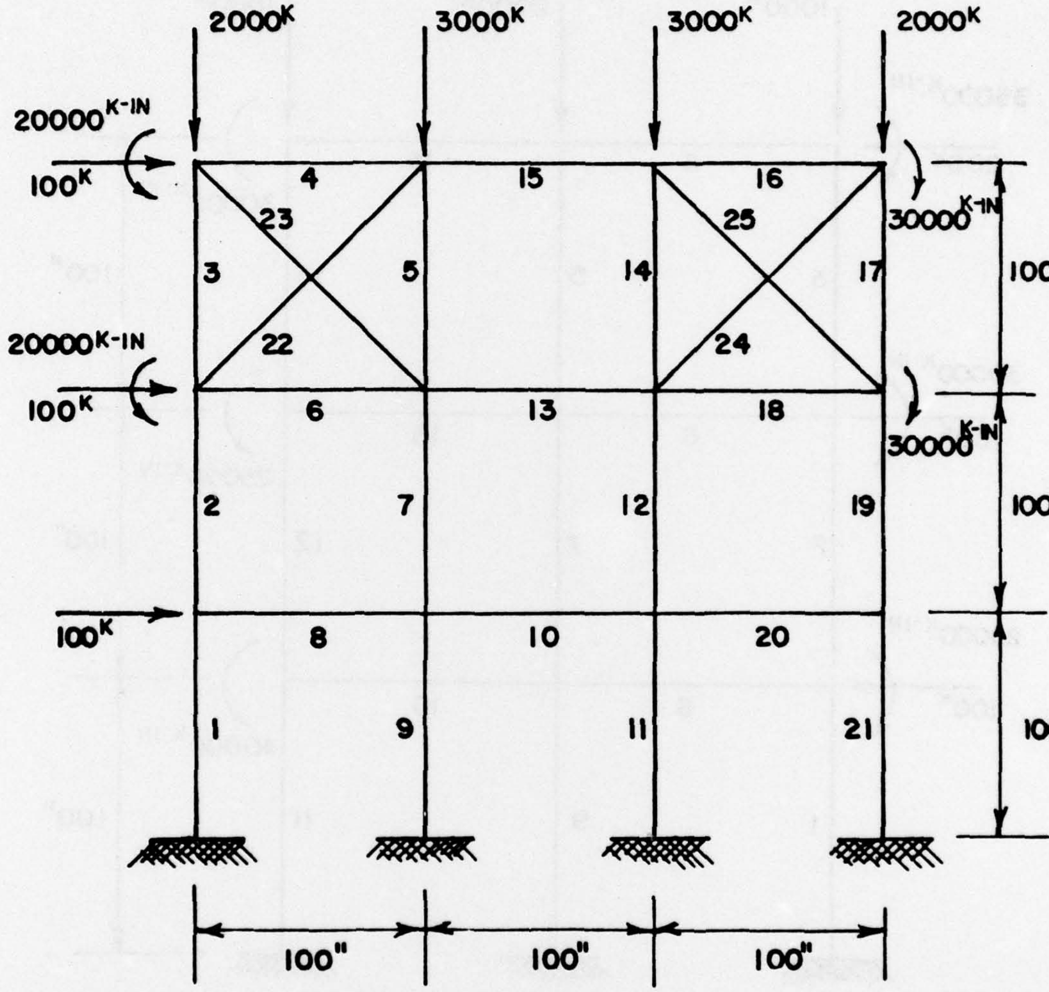

Figure 7: 25 Member Frame (1 kip = 4.45 kN; 1 in = 25.4 mm; 1 kip-in = 113 N·m)

Cycles	Moment of Inertia Ratios		Cross Sectional Areas (Square Inches)			Objective Function
	β_2	β_3	A_1	A_2	A_3	Volume (Cubic Inches)
Start	1.0	1.0	---	---	---	---
1	0.372	0.848	33.55	90.22	39.56	16334.0
2	0.239	0.737	24.08	100.80	32.67	15755.0
3	0.203	0.685	21.11	103.96	30.82	15589.0
4	0.192	0.665	20.15	104.95	30.32	15542.0
5	0.188	0.657	19.84	105.27	30.18	15529.0
6	0.187	0.655	19.74	105.38	30.13	15525.0
Computational Time (Hr:Min:Sec.) = 00:00:36						

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 1A: Results for Three Member Frame Using Linear Programming Technique

Cycles	Moment of Inertia Ratios		Cross Sectional Areas (Square Inches)			Objective Function Volume (Cubic Inches)
	β_2	β_3	A_1	A_2	A_3	
Start	1.0	1.0	90.00	90.00	90.00	---
1	1.05	1.05	94.50	90.00	90.00	27450.0
2	0.223	0.698	24.03	107.63	34.44	16610.0
3	0.198	0.670	21.03	106.12	31.37	15851.0
4	0.190	0.659	20.10	105.65	30.48	15622.0
5	0.188	0.655	19.81	105.50	30.22	15553.0
6	0.187	0.654	19.73	105.45	30.15	15532.0
7	0.187	0.653	19.68	105.43	30.12	15526.0
Computational Time (Hr:Min:Sec) = 00:00:42						

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 1B: Results for Three Member Frame Using Fiacco and McCormick Routine

Cycles	Moment of Inertia Ratios			Cross Sectional Areas (Square Inches)				Objective Function
	β_2	β_3	β_4	A_1	A_2	A_3	A_4	Volume (Cubic Inches)
Start	1.0	1.0	1.0	---	---	---	---	----
2	0.685	1.347	1.507	54.16	79.07	40.21	35.95	25371.0
4	0.592	1.485	1.992	52.12	88.14	35.09	26.16	24027.0
6	0.547	1.526	2.358	50.79	92.91	33.28	21.54	23505.0
8	0.519	1.533	2.608	49.72	95.76	32.42	19.07	23239.0
10	0.501	1.531	2.774	48.92	97.58	31.95	17.63	23087.0
12	0.490	1.527	2.887	48.35	98.76	31.66	16.75	22994.0
14	0.485	1.523	2.963	47.95	99.55	31.47	16.19	22935.0
16	0.476	1.521	3.014	47.68	100.07	31.36	15.82	22896.0
18	0.473	1.518	3.015	47.49	100.41	31.28	15.75	22871.0
Computational Time (Hr:Min:Sec.) = 00:00:45								

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 2A: Results for Four Member Frame Using Linear Programming Technique

Cycles	Moment of Inertia Ratios			Cross Sectional Areas (Square Inches)				Objective Function Volume (Cubic Inches)
	β_2	β_3	β_4	A_1	A_2	A_3	A_4	
Start	1.0	1.0	1.0	75.00	75.00	75.00	75.00	----
2	0.564	1.390	2.032	60.13	106.68	43.27	29.59	28617.0
4	0.514	1.483	2.654	52.09	101.26	35.13	19.63	24575.0
6	0.481	1.518	3.016	48.20	100.28	31.75	15.98	23054.0
8	0.478	1.524	3.050	47.82	100.07	31.37	15.68	22892.0
Computational Time (Hr:Min:Sec.) = 00:03:18								

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 2B: Results for Four Member Frame Using Fiacco and McCormick Routine

Member	Starting Point		Cycle 3		Cycle 6	
	β_j	A_j (Sq. In.)	β_j	A_j (Sq. In.)	β_j	A_j (Sq. In.)
1	--	--	--	128.48	--	129.71
2	1.0	--	13.728	9.36	15.977	8.12
3	1.0	--	3.324	38.66	3.084	42.06
4	1.0	--	4.359	29.49	3.795	34.18
5	1.0	--	0.994	129.20	0.959	135.19
6	1.0	--	0.830	154.72	0.867	149.59
7	1.0	--	4.028	31.90	7.608	17.05
8	1.0	--	1.271	101.11	1.080	120.14
9	1.0	--	7.182	17.89	13.723	9.45
10	1.0	--	25.696	5.00	25.941	5.00
Objective Function	— — — — —		65527.0 in^3		65647.0 in^3	
Computational Time (Hr:Min:Sec) = 00:02:57)						

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 3A; Results for Ten Member Frame Using Linear Programming Technique

Member	Starting Point		Cycle 3		Cycle 7	
	β_j	A_j (Sq. In.)	β_j	A_j (Sq. In.)	β_j	A_j (Sq. In.)
1	--	125.00	--	130.28	--	129.48
2	1.0	125.00	13.226	9.85	15.676	8.26
3	1.0	125.00	3.083	42.26	3.100	41.77
4	1.0	125.00	3.382	38.52	3.456	37.46
5	1.0	125.00	0.965	135.01	0.966	134.04
6	1.0	125.00	0.877	148.63	0.878	147.43
7	1.0	125.00	6.933	18.79	7.758	16.69
8	1.0	125.00	1.077	121.00	1.076	120.31
9	1.0	125.00	9.953	13.09	9.437	13.72
10	1.0	125.00	21.933	5.94	25.896	5.00
Objective Function	— — — — —		67127.0 in^3		66191.0 in^3	
Computational Time (Hr:Min:Sec) = 01:42:32)						

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 3B: Results for Ten Member Frame Using Fiacco and McCormick Routine

Member	CASE I			CASE II			CASE III		
	β_j Initial	β_j Final	A_j Final (Sq.In.)	β_j Initial	β_j Final	A_j Final (Sq.In.)	β_j Initial	β_j Final	A_j Final (Sq.In.)
1	--	--	130.39	--	--	135.08	--	--	135.63
2	1.0	0.997	130.77	5.0	1.037	130.31	10.0	1.042	130.14
3	1.0	0.811	160.74	5.0	0.848	159.34	10.0	0.852	159.23
4	1.0	2.271	57.41	5.0	2.292	58.95	15.0	2.296	59.07
5	1.0	1.550	84.12	5.0	1.619	83.46	10.0	1.627	83.37
6	1.0	12.723	10.25	5.0	13.143	10.28	0.5	13.120	10.34
7	1.0	1.318	98.91	5.0	1.359	99.42	10.0	1.363	99.53
8	1.0	13.995	9.32	5.0	18.452	7.32	10.0	19.233	7.05
9	1.0	1.540	84.69	5.0	1.667	81.04	15.0	1.682	80.64
10	1.0	1.196	109.04	5.0	1.238	109.12	20.0	1.243	109.10
11	1.0	1.145	113.88	5.0	1.175	115.01	10.0	1.177	115.18
12	1.0	1.285	101.47	5.0	1.341	100.74	10.0	1.348	100.61
13	1.0	3.756	34.72	5.0	3.748	36.04	0.5	3.745	36.22
14	1.0	1.220	106.86	5.0	1.271	106.27	10.0	1.276	106.26
15	1.0	1.366	95.47	5.0	1.406	96.08	15.0	1.412	96.09
Objective Function	132805.0 in^3			132843.0 in^3			132846.0 in^3		
Computational Time	00:08:21			00:11:32			00:13:31		

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 4: Results for 15 Member Frame Using Linear Programming Technique

Member	CASE I			CASE II		
	β_j Initial	β_j Final	A_j Final (Sq.In.)	β_j Initial	β_j Final	A_j Final (Sq.In.)
1	--	--	134.43	--	--	138.00
2	1.0	0.907	148.24	5.0	0.929	148.58
3	1.0	0.869	154.64	4.0	0.896	154.08
4	1.0	4.836	27.80	1.0	4.869	28.34
5	1.0	1.045	128.63	6.0	1.070	128.93
6	1.0	26.887	5.00	0.5	27.601	5.00
7	1.0	1.037	129.59	5.0	1.061	130.10
8	1.0	7.242	18.56	1.0	8.782	15.72
9	1.0	0.814	165.17	4.0	0.847	162.97
10	1.0	26.887	5.00	1.0	27.601	5.00
11	1.0	1.114	120.73	0.5	1.141	120.97
12	1.0	1.210	111.11	5.0	1.243	111.06
13	1.0	26.887	5.00	1.0	27.601	5.00
14	1.0	1.095	122.72	6.0	1.131	122.00
15	1.0	26.887	5.00	1.0	27.601	5.00
16	1.0	2.512	53.52	0.5	2.606	52.96
17	1.0	0.705	190.72	3.0	0.720	191.76
18	1.0	26.887	5.00	1.0	27.601	5.00
19	1.0	1.117	120.34	6.0	1.153	119.70
20	1.0	26.887	5.00	1.0	27.601	5.00
21	1.0	1.093	122.94	7.0	1.115	123.74
22	1.0	15.314	8.78	0.5	16.035	8.61
23	1.0	26.887	5.00	1.0	27.601	5.00
24	1.0	26.887	5.00	1.0	27.601	5.00
25	1.0	2.838	47.37	0.5	2.829	48.78
Objective Function	187275.0 in^3			187421.0 in^3		

Note: 1 sq in = 6.45 cm^2; 1 cu in = 16.4 cm^3

Table 5: Results for 25 Member Frame Using Linear Programming Technique

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Optimization, Frames, Linear Programming, Structural Design

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Presented is an optimization technique, using linear programming methods, which can effectively be used to design structural frames. Stress constraints as well as conditions of equilibrium and compatibility are considered. The approach is demonstrated to be more efficient than presently available nonlinear techniques on large problems. Although the method is iterative, a modification of the standard simplex method → next Page

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is used which considerably decreases computational time. A discussion is presented outlining how the technique can be extended to handle additional requirements such multiple loading conditions, non-nodal loads and displacement constraints.

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